Resonant RLC Circuits

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Abstract

The objective of this study was to validate and compare various models for resonant RLC circuits. Models tested were ideal excel calculation, ideal PSpice simulation, real (including internal resistances) excel calculation, real PSpice simulation. The models were compared using the following criteria: accuracy, ease-of-use, time consumption, and price. This study concludes that the ideal PSpice simulation resulted in the maximum efficiency in consideration of the criteria.

1. Introduction

Resonant RLC circuits are often used as filters for communication equipment. Depending on whether or not the circuit is series or parallel, the filter may be a Band-Pass Filter or Band-Stop Filter: allowing only certain frequencies to reach a device. Other applications may include voltage magnification (uses in amplifiers), uses in radio and TV receivers, and induction heating. RLC circuits contain a resistor (R), inductor (L), and capacitor (C). When a voltage is introduced to the three elements, each element effects the voltage in various ways.

The objective of this study was to perform an analysis to determine the accuracy of various RLC circuit models and validate their effect on the gain at various frequencies. Gain (A), the quotient of output voltage over input voltage, is used as a measure of how the RLC elements effect the voltage.

1.1 Models

The use of models allows for a prediction of the circuits response without creating and testing the circuit physically. Models are assessed by their accuracy, simplicity, ease of use, and time required. In this study, two models are tested for both series and parallel resonant RLC circuits.

1.2 Resonance

A resonant circuit is one containing elements (typically inductors and capacitors) that allow for different outputs as a function of the circuit elements. Resonant circuits contain a peak or minimum frequency value that is considered the circuits resonant frequency (ω_0). In resonant RLC circuits, ω_0 may be calculated using the following equation:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

2. Experimental Procedure

To find experimental values of gain within resonant RLC circuits, two RLC circuits were built and given a constant input voltage. The output voltage of each circuit was then tested at varying frequencies to plot values of gain as a function of frequency. The setup and overall procedure are outlined in the sections to follow.

2.1 Equipment Used

To test the models of series and parallel resonant RLC circuits, accurate values for the following must be found: inductor, capacitor, load resistance, inductive resistance (R_{ind}), and source resistance (R_{src}). The R, L, C, and R_{ind} values are found using an impedance meter.
A GW Instek Function Generator is then used to create a sinusoidal function to be input into circuit. To accurately measure the input and output voltage, a Tektronix Oscilloscope is used.

2.2 Experimental Setup

To begin the experiment, RLC and internal resistance values were calculated. The internal resistance of the function generator was calculated by measuring the open-circuit voltage (no applied load), a resistor value, and the closed-circuit voltage (parallel to an attached resistor). The voltage divider equation below was then used to calculate the source resistance (R_{src}).

$$V_{out} = \left(\frac{R_L}{R_L + R_{src}}\right) * V_{ac}$$

The RLC and inductor resistance values were then measured using an impedance meter.

To calculate experimental gain, output voltage must be measured at varying frequencies for a constant input voltage. First, the function generator was set to output a 2V amplitude and 4V peak-to-peak value. The function generator was connected to the oscilloscope with no load: amplitude and peak-to-peak values are more accurately measured by the oscilloscope. The amplitude of the function generator was then changed (possibly to non-2V values) to get a 2V amplitude on the oscilloscope. Once the voltage source was at 2V, the RLC series resonant circuit was wired up. For a schematic of the RLC series resonant circuit, see Figure 1 in section 2.3. The second channel of the oscilloscope was then attached in parallel to the resistive load to measure output voltage. The frequency on the function generator was then varied between 1 KHz and 1MHz. As frequency was varied, the frequency and output voltage values were measured.

The procedure for the RLC series circuit was then repeated for the resonant RLC parallel circuit. Once values for input voltage, output voltage, and frequency were measured, gain was calculated and compared to model values. The model types and methods are discussed in the next section.

2.3 Model Selection and Tests

Models tested for the accuracy and effectiveness of resonant RLC circuits were, first of all, “ideal” and “real.” Ideal refers to a setup with the AC voltage source and inductor are modeled without an internal resistance. Real refers to a setup with a source resistance and inductive resistance. See schematics for ideal and real series resonant RLC circuits in Figures 1 and 2 below.

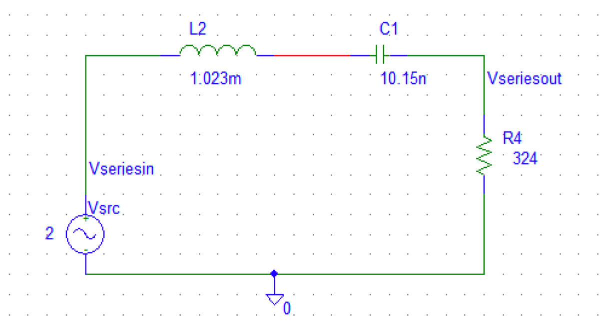

Figure 1: Ideal Series Resonant RLC Circuit Schematic

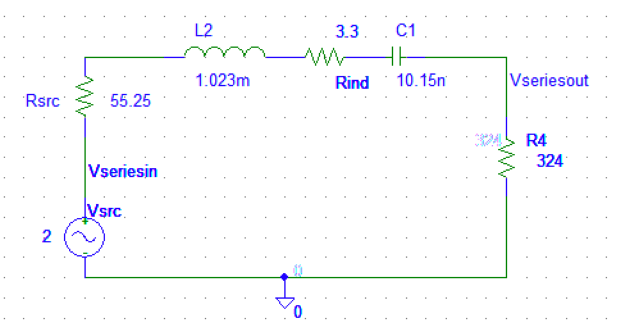

Figure 2: Real Series Resonant RLC Circuit Schematic

Models for parallel resonant RLC circuits were also tested for accuracy and effectiveness. The setups were similar to series, but with the inductor and capacitor put in parallel (with the resistor in series with both). In the real schematic, the source resistance and inductor resistance are added in series with their corresponding components. See Figures 3 and 4 below for schematics.

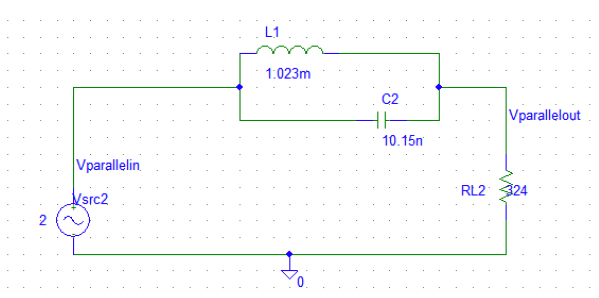

Figure 3: Ideal Parallel Resonant RLC Circuit

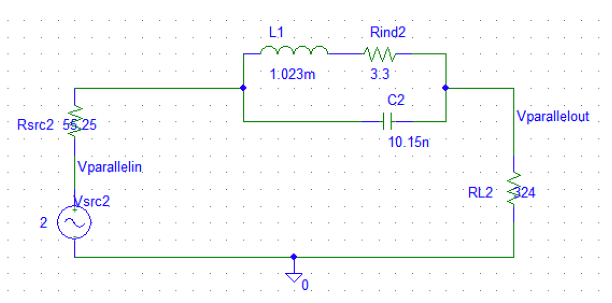

Figure 4: Real Parallel Resonant RLC Circuit

Both ideal and real RLC circuits at specified R, L, and C values were then simulated in PSpice to find the given input voltage and output voltage. The quotient of output over input was then used to calculate the PSpice ideal gain and Series PSpice real gain for RLC in series.

Two other models were created using Excel calculations. Excel calculations were done by finding the equivalent impedance of the RLC circuit elements. The equivalent impedances vary based on frequency: which is converted into angular frequency (ω) using the equation:

$$\omega = 2\pi f \qquad where\ f\ is\ frequency\ in\ Hz$$

The gain is then calculated by taking the absolute value of the quotient of the load resistance value over the equivalent impedance:

$$A = \left|\frac{V_0}{V_i}\right| = \left|\frac{R_L}{Z_{total}}\right|$$

For full calculations for excel ideal series gain values, see Figure A1 in Appendix A. For full excel work, see Figure B1 in Appendix B. Calculations for real series gain, ideal parallel gain, and real parallel gain only vary from the given calculation in the equivalent impedance equations. Results of the gain values of various models were then compared to the experimental gain values.

3. Experimental Results

3.1 Recorded Data

Measured experimental values for R, L, C, inductive resistance, and source resistance are shown in Table A1 in Appendix A. Gain and frequency values tabulated for the all four series RLC models may be seen in Table B1 in Appendix B. Tabulated values for frequency and gain in all four parallel RLC models may be seen in Table B2 in Appendix B.

The gain values for the four RLC series models are compared in Figure 5 below.

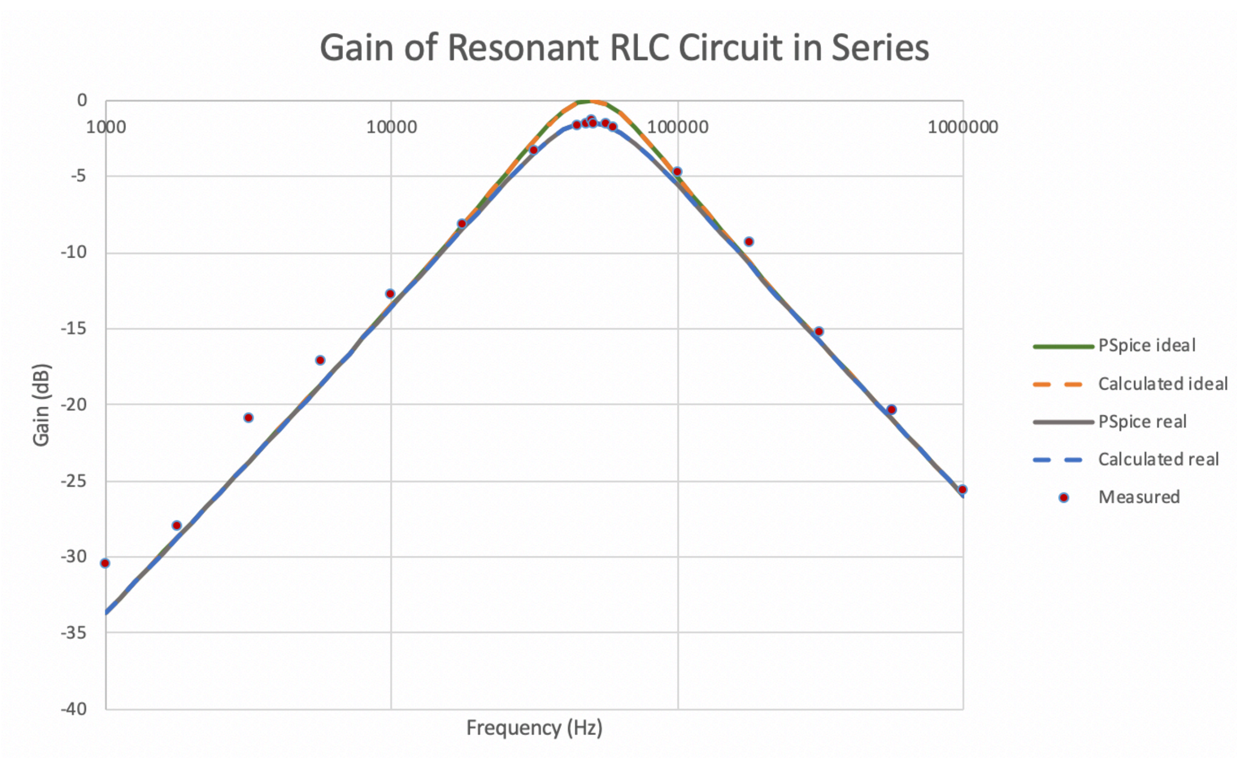

Figure 5: Gain values at varying frequencies for models of the resonant RLC series circuit are plotted. Note that the frequency (x-axis) values are placed on a log scale. Models are plotted as lines; measured values are plotted as points.

The gain values for the four RLC parallel models are plotted in Figure 6 below.

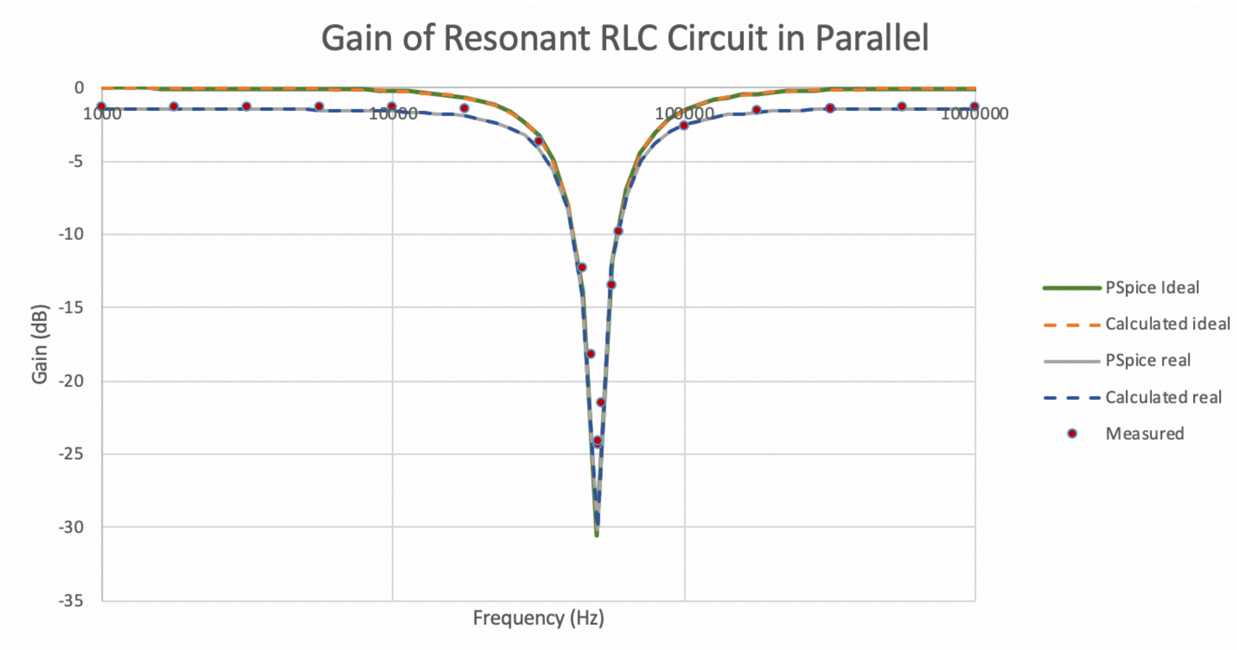

Figure 6: Gain values at varying frequencies for models of the resonant RLC parallel circuit are plotted. Note that the frequency (x-axis) values are placed on a log scale. Models are plotted as lines; measured values are plotted as points.

3.2 Data Analysis

The resonant RLC series gain models are plotted as a function of frequency in Figure 5 on the previous page. Note that the model curves begin low at low frequencies, hit the peak at some median value frequency, then ends low at high frequencies. The peak frequency is referred to resonant frequency (ω_0), at this point, the gain is maximized. In Figure 5, also note that the ideal values (both calculated and simulated) are above the real values and measured values. The difference in gain is due to the ideal model ignoring internal resistance of the inductor and source. According to the data collected, the real data correlates well with the measured data. The gain values of the measured are close and the gain values of the real curves.

The circuit peaking at a median frequency is to be expected. This is because at low frequencies, the inductor impedance is zero, and the capacitor impedance is infinity. At frequencies in the high extrema, the impedance is infinite, and the capacitor value is zero. The total impedance at the extrema is maximized as a result. At the resonant frequency, the impedances of the inductor and capacitor cancel: their impedance values are equal and cancel. The cancellation of the LC components results in a peak in Gain.

The resonant RLC parallel gain models are plotted as a function of frequency in Figure 6 on the previous page. Note that the model curves have high gain at low and high frequencies. The gain changes abruptly in the mid-range of frequency. The "trough" of the curve is the resonant frequency for the parallel circuit. Note again that the ideal models have higher gain at instantaneous frequencies than the real models and measured values. This is also due to the lack of consideration for the internal resistances. The accuracy of the real models is better than that of the ideal models.

The circuit trough at median frequencies is also to be expected. Placing the inductor and capacitor in parallel essentially reverses their relationship with one another. At the frequency extrema, the impedances are equal and cancel each other out. The resonant frequency in the parallel RLC case is the frequency at which the LC total impedance is maximized, resulting is a significant drop in gain values.

As a result of the data in both series and parallel, predictions to the level of filtration (or gain value) at various frequencies may be fairly accurately made. At frequencies around the resonant frequency, the gain will be around the maximum or minimum value. At frequencies far away from the resonant frequency, the gain will vary significantly from the gain at resonant value.

3.3 Error Values

Error values were found using the frequency at which each model's gain was at its respective peak. For both series and parallel, the ideal and real had the same resonant frequency, only varying in gain values at the critical point. The errors for all models were within 2% of the measured values.

Error values may be better found using the change in gain; however, the measured gain values for this lab were not accurately at the peak or minimum gain value. From observation, however, the correlation between the real models is much more accurate to the experimentally measured values. As a result of the visual observation, this experiment yields that the real models are most accurate.

4. Conclusion

The models tested in this experiment were judged based on the accuracy, ease-of-use, cost, and time consumed. The four models in question – calculated ideal, calculated real, PSpice ideal, and PSpice real – are all relatively accurate for most purposes. The cost of the calculated models is free but are significantly more time consuming and difficult to use. The imaginary value calculations complicate work in Excel or Mathcad.

Sources of error within this experiment may be present due to human error. Human error may occur when building circuits or measuring values. Human error may also occur during hand or excel calculations. For the best accuracy, PSpice has the smallest room for human error.

Based on my calculations and criteria, the best model for a resonant RLC circuit is to simulate a real circuit in PSpice. The real PSpice is very accurate, among the easiest of the models to use, and relatively low amount of time consumption; however, this model requires a purchase of PSpice software. This report concludes that the ease of use and time consumption outweigh the cost difference.

References

Resonance Measurements and Reporting Guidelines from Calvin University's Engineering Department

"Resonant RLC Circuits Working and Application." *Electronics Project Focus*, 13 Nov. 2019, www.elprocus.com/guide-on-resonant-rlc-circuits-working-and-application/.

Zekavat, Reza. *Electrical Engineering: Concepts and Applications*. Pearson, 2013.

Appendices

Appendix A – Measured Data and Calculations

Table A1: Measured values for load resistance, inductance, capacitance, and internal resistances

R (Ω)	L (mH)	C (nF)	R_{src} (Ω)	R_{ind} (Ω)
324	1.023	10.15	55.25	3.3

$Calculations\ for\ ideal\ resonant\ RLC\ series\ circuit:$

$f = 1000\ Hz \quad \rightarrow \quad \omega = 2\pi f = 2000\pi$

$Z_L = j\omega L = j6.4277\ \Omega \qquad Z_C = -\frac{1}{j\omega C} = -j15757.9\ \Omega. \qquad Z_R = R = 324\ \Omega$

$$\sum Z = 324 - j15673.86 \qquad \rightarrow \qquad 15677.21 < -88.8°$$

$$A = \frac{R_L}{Z_{tot}} = \frac{324 < 0°}{15677.21 < -88.8°} = .020667 < 88.8°$$

A = .0004328 + j.0206624

$$|A| = .020667 \quad \rightarrow \quad 20\log|A| = -33.6944$$

$Repeat\ Calculations\ in\ Excel\ for\ All\ Values\ of\ Frequency.$

Also Repeat Calculations for all models (real series, ideal parallel, real parallel) varying only calculation of total impedance.

Figure A1: Calculations for ideal resonant RLC series circuit

Appendix B – Model Values

Table B1: Values for various models of the resonant **series** RLC circuit. Gain values for PSpice Ideal, Excel Ideal, PSpice Real (Actual), and Excel Real are given at various frequencies. Units for Frequency are Hz and Gain is unitless (V/V).

Frequency	Pspice Gain Real	Excel Gain Ideal	PSPICE Gain Actual	Excel Gain Actual
1000	-33.69447573	-33.69447568	-33.7	-33.69520662
1122.01845	-32.69403414	-32.69403411	-32.69	-32.69495438
1258.92541	-31.69347848	-31.69347828	-31.69	-31.69463694
1412.53754	-30.69277845	-30.69277864	-30.69	-30.6942375
1584.89319	-29.69189782	-29.69189805	-29.69	-29.69373494
1778.27941	-28.69078958	-28.69078975	-28.69	-28.69310272
1995.26231	-27.6893946	-27.68939496	-27.69	-27.69230754
2238.72114	-26.68763981	-26.68763977	-26.69	-26.69130767
2511.88643	-25.68543109	-25.68543133	-25.69	-25.69005078
2818.38293	-24.68265306	-24.68265297	-24.69	-24.68847143
3162.27766	-23.67915813	-23.67915826	-23.69	-23.68648789
3548.13389	-22.67476393	-22.6747635	-22.68	-22.68399827
3981.07171	-21.66923828	-21.66923849	-21.68	-21.68087595
4466.83592	-20.66229487	-20.66229513	-20.68	-20.67696415
5011.87234	-19.65357371	-19.65357344	-19.67	-19.67206961
5623.41325	-18.64262487	-18.6426246	-18.67	-18.66595567
6309.57344	-17.62889046	-17.62889062	-17.66	-17.65833497
7079.45784	-16.61168027	-16.61168045	-16.65	-16.64886269
7943.28235	-15.59014292	-15.59014267	-15.64	-15.63713189
8912.50938	-14.56323572	-14.56323582	-14.62	-14.62267396
10000	-13.52969911	-13.52969892	-13.6	-13.6049695
11220.1845	-12.48802778	-12.48802795	-12.58	-12.58347884
12589.2541	-11.4364697	-11.43646964	-11.56	-11.55770829
14125.3754	-10.37305467	-10.37305507	-10.53	-10.5273393
15848.9319	-9.295715789	-9.295715459	-9.492	-9.492466861
17782.7941	-8.202560672	-8.202560727	-8.454	-8.454023789
19952.6231	-7.092471147	-7.092471044	-7.415	-7.414515099
22387.2114	-5.966277402	-5.966277566	-6.379	-6.379252755
25118.8643	-4.829019454	-4.82901969	-5.358	-5.358350068
28183.8293	-3.694060384	-3.694060377	-4.37	-4.369733267
31622.7766	-2.590020022	-2.590019944	-3.443	-3.443136244
35481.3389	-1.570670529	-1.570670533	-2.624	-2.624005132
39810.7171	-0.723821512	-0.723821716	-1.974	-1.974006459
44668.3592	-0.165841716	-0.165841448	-1.562	-1.562463989
50118.7234	-0.003565102	-0.003565266	-1.445	-1.445421659
56234.1325	-0.273576683	-0.273576748	-1.641	-1.640837464
63095.7344	-0.916455729	-0.916455827	-2.119	-2.119250915
70794.5784	-1.817374143	-1.81737403	-2.819	-2.818748142
79432.8235	-2.865121864	-2.865121752	-3.67	-3.670433954
89125.0938	-3.980925461	-3.980925513	-4.616	-4.616416782
100000	-5.118424617	-5.118424765	-5.616	-5.615509407
112201.845	-6.253713939	-6.253713807	-6.641	-6.64138623
125892.541	-7.376073799	-7.376074001	-7.678	-7.678451922
141253.754	-8.481835169	-8.481834831	-8.718	-8.718054518
158489.319	-9.570821071	-9.570820956	-9.756	-9.755749003
177827.941	-10.64442986	-10.64442998	-10.79	-10.78952291
199526.231	-11.70463763	-11.70463751	-11.82	-11.81871086
223872.114	-12.75350485	-12.75350499	-12.84	-12.84335402
251188.643	-13.79295327	-13.79295327	-13.86	-13.86383266
281838.293	-14.82467348	-14.82467337	-14.88	-14.88066129
316227.766	-15.85010606	-15.85010602	-15.89	-15.89437892
354813.389	-16.87045387	-16.87045384	-16.91	-16.90549395
398107.171	-17.8867069	-17.88670714	-17.91	-17.91445982
446683.592	-18.8996739	-18.89967386	-18.92	-18.92166747
501187.234	-19.91000909	-19.91000878	-19.93	-19.92744655
562341.325	-20.91824018	-20.91824004	-20.93	-20.93207088
630957.344	-21.92479232	-21.92479211	-21.94	-21.93576538
707945.784	-22.93000554	-22.93000522	-22.94	-22.93871341
794328.235	-23.93415132	-23.93415155	-23.94	-23.94106354
891250.938	-24.93744868	-24.9374485	-24.94	-24.94293563
1000000	-25.94006934	-25.94006951	-25.94	-25.94442602

Table B2: Values for various models of the resonant **parallel** RLC circuit. Gain values for PSpice Ideal, Excel Ideal, PSpice Real (Actual), and Excel Real are given at various frequencies. Units for Frequency are Hz and Gain is unitless (V/V).

Frequency	Pspice Gain Ideal	Excel Gain Ideal	PSPICE Gain Actual	Excel Gain Actual
1000	-0.001710413	-0.001710314	-1.444152083	-1.444152015
1122.01845	-0.002153434	-0.002153505	-1.444486045	-1.444485813
1258.92541	-0.002711597	-0.002711654	-1.444906384	-1.444906202
1412.53754	-0.003414689	-0.003414643	-1.445435742	-1.445435699
1584.89319	-0.004300304	-0.004300165	-1.446102815	-1.446102706
1778.27941	-0.005415963	-0.005415782	-1.446943336	-1.446943066
1995.26231	-0.006821749	-0.006821546	-1.448001904	-1.448002046
2238.72114	-0.0085933	-0.008593338	-1.449336975	-1.449336855
2511.88643	-0.01082705	-0.010827128	-1.45101973	-1.451019872
2818.38293	-0.013644398	-0.013644441	-1.453142527	-1.453142779
3162.27766	-0.017199131	-0.017199384	-1.455822027	-1.455821895
3548.13389	-0.021687734	-0.021687751	-1.459205256	-1.459205087
3981.07171	-0.027358997	-0.027358868	-1.46348065	-1.463480788
4466.83592	-0.034531025	-0.034531148	-1.468889849	-1.468889864
5011.87234	-0.043612734	-0.043612689	-1.475741387	-1.475741374
5623.41325	-0.055128801	-0.055128854	-1.484433755	-1.484433753
6309.57344	-0.069759408	-0.069759657	-1.495483586	-1.495483653
7079.45784	-0.088391028	-0.088391207	-1.509566014	-1.509565821
7943.28235	-0.11218772	-0.112187625	-1.527569162	-1.527569208
8912.50938	-0.142693359	-0.142693487	-1.550677278	-1.550677457
10000	-0.181982656	-0.181982708	-1.580486688	-1.580486897
11220.1845	-0.232879703	-0.232879796	-1.619183849	-1.619183692
12589.2541	-0.299296617	-0.29929664	-1.669816686	-1.669816799
14125.3754	-0.386758516	-0.386758653	-1.736731008	-1.736730699
15848.9319	-0.503250038	-0.503249986	-1.826273348	-1.8262731
17782.7941	-0.660612838	-0.660612808	-1.947992499	-1.947992501
19952.6231	-0.87694036	-0.876940371	-2.116742549	-2.116742413
22387.2114	-1.180817115	-1.180817062	-2.356536187	-2.356536093
25118.8643	-1.619131567	-1.619131559	-2.707944582	-2.707944265
28183.8293	-2.272150703	-2.272150971	-3.243061737	-3.243061566
31622.7766	-3.284453765	-3.284453665	-4.097768787	-4.097768824
35481.3389	-4.935464539	-4.935464318	-5.547905575	-5.547905875
39810.7171	-7.840995451	-7.840995278	-8.224945382	-8.224945142
44668.3592	-13.92374129	-13.92374145	-14.10086987	-14.10086998
50118.7234	-30.50539947	-30.50539941	-30.09471138	-30.09471151
56234.1325	-11.81194588	-11.81194612	-11.98580341	-11.9858036
63095.7344	-6.922818662	-6.922818843	-7.285103273	-7.285103447
70794.5784	-4.431017711	-4.431017516	-5.004117022	-5.004117034
79432.8235	-2.981282128	-2.981282185	-3.740540353	-3.740540341
89125.0938	-2.079512239	-2.079512029	-2.9862766	-2.986276256
100000	-1.491411525	-1.491411494	-2.509927645	-2.509927864
112201.845	-1.093165974	-1.093166077	-2.195127742	-2.195127995
125892.541	-0.815062168	-0.81506196	-1.979300109	-1.979300288
141253.754	-0.615907353	-0.615907358	-1.826876244	-1.82687609
158489.319	-0.470339186	-0.470339076	-1.716636395	-1.716636268
177827.941	-0.362159505	-0.362159746	-1.635372384	-1.635372157
199526.231	-0.280683937	-0.280684159	-1.57454891	-1.574548804
223872.114	-0.218658075	-0.218657879	-1.528468556	-1.52846841
251188.643	-0.171029704	-0.171029822	-1.493218118	-1.493217891
281838.293	-0.134204866	-0.13420499	-1.466043201	-1.466043328
316227.766	-0.105576154	-0.10557591	-1.444965301	-1.444965571
354813.389	-0.083220763	-0.083220653	-1.428536683	-1.428536752
398107.171	-0.065703453	-0.065703205	-1.415681511	-1.415681675
446683.592	-0.05193842	-0.051938348	-1.40559176	-1.405591844
501187.234	-0.041097966	-0.041098191	-1.397653107	-1.397652965
562341.325	-0.032546073	-0.032546231	-1.391394158	-1.391394298
630957.344	-0.025789996	-0.02578999	-1.386452805	-1.38645258
707945.784	-0.020446469	-0.020446447	-1.38254582	-1.382545889
794328.235	-0.016216488	-0.016216464	-1.379454542	-1.379454415
891250.938	-0.012865834	-0.012865619	-1.377006053	-1.377006141
1000000	-0.010209827	-0.010209709	-1.375066001	-1.375066048

Appendix C – Experimental Values

Table C1: Experimental Output Values at varying frequencies for experimental resonant **series** RLC circuit. Gain is calculated from input and output voltage values.

Frequency (Hz)	Input Voltage (Vp)	Output Voltage (Vp)	Vout/Vin	Gain
1000	2	0.06	0.03	-30.458
1778	2	0.08	0.04	-27.959
3162	2	0.18	0.09	-20.915
5623	2	0.28	0.14	-17.077
10000	2	0.46	0.23	-12.765
17783	2	0.78	0.39	-8.179
31623	2	1.36	0.68	-3.350
45000	2	1.64	0.82	-1.724
48000	2	1.66	0.83	-1.618
50500	2	1.72	0.86	-1.310
51000	2	1.68	0.84	-1.514
56234	2	1.68	0.84	-1.514
60000	2	1.62	0.81	-1.830
100000	2	1.16	0.58	-4.731
177828	2	0.68	0.34	-9.370
316228	2	0.344	0.172	-15.289
562341	2	0.192	0.096	-20.355
1000000	2	0.104	0.052	-25.680

Table C2: Experimental Output Values at varying frequencies for experimental resonant **parallel** RLC circuit. Gain is calculated from input and output voltage values.

Frequency (Hz)	Input Voltage (Vp)	Output Voltage (V)	Vout/Vin	Gain
1000	2	1.7	0.85	-1.412
1778	2	1.7	0.85	-1.412
3162	2	1.7	0.85	-1.412
5623	2	1.7	0.85	-1.412
10000	2	1.72	0.86	-1.310
17783	2	1.68	0.84	-1.514
31623	2	1.3	0.65	-3.742
45000	2	0.48	0.24	-12.396
48000	2	0.244	0.122	-18.273
50500	2	0.12	0.06	-24.437
51000	2	0.124	0.062	-24.152
52000	2	0.166	0.083	-21.618
56234	2	0.424	0.212	-13.473
60000	2	0.64	0.32	-9.897
100000	2	1.48	0.74	-2.615
177828	2	1.66	0.83	-1.618
316228	2	1.7	0.85	-1.412
562341	2	1.72	0.86	-1.310
1000000	2	1.72	0.86	-1.310